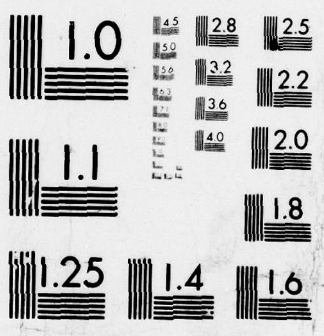
AD-A074 743 ROYAL AIRCRAFT ESTABLISHMENT FARNBOROUGH (ENGLAND) F/G 20/11
LOAD DISTRIBUTION BETWEEN HARMONIC DRIVE TEETH (O RASPREDELENII--ETC(U)
AUG 77 Y P FUKS, V A FINOGENOV

UNCLASSIFIED RAE LIBRARY TRANS-1929 DRIC-BR-62919 NL

1 OF 1

AD
A074 743

END
DATE
FILMED
11-79
DDC



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

UNLIMITED

Trans 1929

ROYAL AIRCRAFT ESTABLISHMENT

*

Library Translation 1929

August 1977

LOAD DISTIBUTION BETWEEN HARMONIC DRIVE TEETH

by

Yu.P. Fuks
V.A. Finogenov

*

Procurement Executive, Ministry of Defence
Farnborough, Hants

UNLIMITED

Translations in this series are available from:

THE R.A.E. LIBRARY
Q.4 BUILDING
R.A.E. FARNBOROUGH
HANTS

New translations are announced monthly in:

LIBRARY LIST C
RAE REPORTS AND PUBLICATIONS

Procurement Executive, Ministry of Defence

DEFENCE RESEARCH INFORMATION CENTRE

St. Mary Cray, Orpington, Kent BR5 3RE

Telex 896866 Telephone. Orpington 32111 ext. 107

The Defence Documentation Centre
USA
through ONR

Your reference.

Our reference
HO/75/032

Date
14/6/78.

REPORT RAE LIB TRANS

TITLE: LOAD DISTRIBUTION BETWEEN HARMONIC DRIVE TEETH

We enclose one hard copy of the above report and would be pleased if you would copy and distribute to the addressees listed on attached list stating 'forwarded on the recommendation of MOD(PE) Defence Research Information Centre, UK'. This is in line with your policy of only one copy of each report to be forwarded to DDC through ONR LONDON.

P Roberts

P ROBERTS (MRS)
for Head/DRIC

Enc.

1 - Library of Congress.

UDC 621.83.05 : 621.23 : 620.172.2

(12) 76

ROYAL AIRCRAFT ESTABLISHMENT

(14) RAE Library Translation 1929 (11) Aug 77

Received for printing 11 August 1977

(6)

LOAD DISTRIBUTION BETWEEN HARMONIC DRIVE TEETH

(O RASPREDELENII NAGRUZKI MEZHDU ZUB'YAMI VOLDOVYKH PEREDACH),

by

(10) Yu. P./Fuks

V. A./Finogenov

Izv. Vyssh. Uchebn. Zaved. Mashinostr, 12, 23-28 (1971)

(21) Trans. of Izvestiya Vysshikh Uchebnykh Zavedenii Mashinostr, v12 p23-28 1971, by J. W. Palmer

Translator
J.W. Palmer

Translation editor
K. James

(18) DRIC

(19) BR-62919

AUTHORS' SUMMARY

This paper gives a method for determining the load acting on the teeth of harmonic gears.

310450

JOB

78 07 17 049

LIST OF CONTENTS

Let us examine an harmonic tooth gear (Fig 1) consisting of an immobile rigid gear Ж (r) , a rotating flexible gear Г(G) and a cam generator H(N) . Let us assume that the profile of the generator ensures in a static state and under load contact of all the teeth within the limits of the sector AOD where the teeth of the flexible gear are located in the cavities of the teeth of the rigid gear. The teeth of the rigid gear are of trapezoidal shape having a profile angle α (Fig 2). The difference in the load on individual pairs of teeth is caused by the fact that their instantaneous velocities differ. Consequently, the distribution of the load may be shown by analysis of the kinematics of the teeth.

It was shown in Ref 1 that the motion of the teeth of the rotating flexible gear is the result of the summation of two movements: loop-shaped local movement with velocities of V_y and V_z - caused by displacement of the deformation wave, and the transferable peripheral motion with a velocity of V_{per_G} - together with the flexible gear. The sum of the peripheral velocities of these two movements

$$\overline{V}_{\tau_i} = \overline{V}_{per_{G_i}} + \overline{V}_{y_i} \quad . \tag{1}$$

The vector $\overline{V}_{\tau_i}$ is directed to the side opposite the rotation of the generator and in the peripheral direction the tooth of the flexible gear moves away from the tooth of the rigid gear. Besides the peripheral displacement with velocity $\overline{V}_{\tau_i}$ the tooth of the flexible gear is displaced in a radial direction also with a velocity $\overline{V}_{z_i}$. The cumulative velocity of the tooth of the flexible gear $\overline{V}_i^* = \overline{V}_{\tau_i} + \overline{V}_{z_i}$ (Fig 2). In order to retain the contact of the teeth it is necessary[1,2] that the tooth of the rigid gear should have the shape of a wedge with a profile angle α_i

$$tg\alpha_i = \frac{V_{\tau_i}}{V_{z_i}} \ , \tag{2}$$

whence

$$V_{\tau_i} = V_{z_i} tg\alpha_i \tag{3}$$

here α_i is the angle which ensures the kinematic closure of the teeth (in what follows α_i is called the kinematic angle).

Let us examine the possible relationships between the kinematic angle α_i and the profile angle α (Fig 2): (1) $\alpha_i > \alpha$. The velocity $\overline{V}_{\tau_{\alpha_i}} > \overline{V}_{\tau_\alpha}$ (*cf.* relationship (3)), here $\overline{V}_{\tau_{\alpha_i}}$ is the velocity necessary for kinematic closing: $\overline{V}_{\tau_\alpha}$ is the actual velocity. The tooth of the flexible gear moves away from the profile of the tooth of the rigid gear: the contact is broken; (2) $\alpha_i = \alpha$. The velocity $\overline{V}_{\tau_{\alpha_i}} = \overline{V}_{\tau_\alpha}$ and the tooth of the flexible gear slips, without being deformed, over the profile of the tooth of the rigid gear; (3) $\alpha_i < \alpha$. The velocity $\overline{V}_{\tau_{\alpha_i}} < \overline{V}_{\tau_\alpha}$, consequently, an additional peripheral velocity is transmitted to the tooth of the flexible gear as a result of its deformation.

$$V_{elast} = V_{\tau_\alpha} - V_{\tau_{\alpha_i}} = V_{z_i} \mathrm{tg}\alpha - V_{z_i} \mathrm{tg}\alpha_i = V_{z_i} (\mathrm{tg}\alpha - \mathrm{tg}\alpha_i) \qquad (4)$$

where V_{elast} is the velocity of the elastic deformation resulting from the kinematics. The loading of the tooth is accompanied by its elastic deformation. If there is no deformation ($\alpha_i \geqslant \alpha$), then the useful load is also not transmitted by the teeth. The condition for transmission of the load is $\alpha_i < \alpha$.

If we consider the velocities which make up formula (4), as mean ones over the small time-interval Δt , equal to the time of rotation of the generator through one angular step $\left(\Delta t = \frac{2\pi}{z_G \omega_H}\right)$, and multiply all the terms of the expression (4) by Δt , we then obtain equation (4) in the displacements

$$S_{elast} = S_{z_i} (\mathrm{tg}\alpha - \mathrm{tg}\alpha_i) \ , \qquad (5)$$

where $S_{elast} = V_{elast} \Delta t$ is the elastic displacement of the tooth of the flexible gear in a peripheral direction (disregarding the deformation of the system under load); $S_{z_i} = V_{z_i} \Delta t$ is the radial displacement of the tooth of the flexible gear in the time Δt . The peripheral force $P(\varphi)$ acting on the tooth, is proportional to its deformation

$$P(\varphi) = h_i^* S_{z_i} (\mathrm{tg}\alpha - \mathrm{tg}\alpha_i) = h_i^* S_{z_i} f(\alpha_i) \qquad (6)$$

where h_i^* is the coefficient of proportionality (scale coefficient); $f(\alpha)_i = (\mathrm{tg}\alpha - \mathrm{tg}\alpha_i)$ is the kinematic function.

The magnitude of the angle α_i may be calculated from formula (2) by substituting the corresponding values V_{τ_i} and V_{z_i} (Ref 1). We determine the value of the function $f(\alpha)$ at two characteristic points[1] : (1) at the point $A(\varphi = 0$, Fig 1) , vector $V_{\tau_A} = 0$ and $tg\alpha_A = 0$. Whence $f(\alpha)_A = tg\alpha_i$, (2) at the point $C(\varphi \approx 45^o)$ the vector $V_{\tau_C} = V_{per_{G_C}} - V_{y_C} = V_{per_{G_C}} - 0 = V_{per_{G_C}} \approx \omega_H\Delta$, where Δ is the maximum deformation of the flexible gear (measured along the large axis of the generator, Fig 1).

In the case of radial deformation of the flexible gear according to the law $\omega = \Delta \cos 2\varphi$ we obtain[1] $V_{z_C} \approx 2\omega_H\Delta$, whence $tg\alpha_C = \frac{V_{\tau_C}}{V_{z_C}} \approx \frac{\omega_H\Delta}{2\omega_H\Delta} = 0.5$. The function $f(\alpha)_C = (tg\alpha - 0.5)$.

By putting at the point D $f(\alpha)_D \approx 0$, as a first approximation it can be assumed that the function $f(\alpha)$ in the ranges $\varphi = 0 - 45^o$ and $\varphi = 45 - \varphi^o_{law}$ varies according to a linear law.

The magnitude of the radial displacement S_{z_i} is determined by the shape of the deformed flexible gear. The coefficient h^*_i is dependent upon the reduced stiffness of the connecting pairs of teeth. In order to estimate the relationships of stiffness of the connected pairs of teeth y_Σ we determined theoretically the values of the components $y_\Sigma = y_1 + y_2 + y_G + y_{oth}$, where y_1, y_2 is the stiffness of the teeth of the flexible and rigid gears respectively; y_G is the stiffness of the flexible gear; y_{oth} is the stiffness of the generator and other units.

These calculations showed that for five-modulus steel teeth ($m \leq 1$) and a flexible gear ≈ 3 m thick the stiffness of the rim of the flexible gear $y_G \gg (y_1 + y_2)$ approximately by an order of two, therefore the variation of the magnitude $(y_1 + y_2)$ may be disregarded. Therefore, the reduced stiffness (rigidity) for all connected pairs of teeth is practically identical. Consequently, $h^*_i = h^* = const$. The magnitude of the coefficient is determined from the condition

$$\sum_{i=1}^{i=n} M_0[P(\varphi)] = M_{tor}$$

whence

$$h^* = \frac{2M_{tor}}{uD_r \sum_{i=1}^{i=n} S_{z_i} f(\alpha)_i}$$

here M_{tor} is the torque on the gear, kg m ; u is the number of deformation waves; D_r is the diameter of the initial circumference of the rigid gear, cm ; n is the number of pairs of teeth coupled in the zone of active engagement, $n = \frac{\varphi_{eng} Z_r}{2\pi}$, where φ_{eng} is the arc of engagement, Z_r is the number of teeth of the rigid gear. In order to estimate the relationship (6) a calculation was made of the values $P(\varphi)$ of the double-wave gear[2] having the parameters: $\alpha = 30^\circ$; $m = 0.8$ mm ; $f_0 = 0.8$; $c_0 = 0.2$; $z_G = 200$; $z_r = 202$; $\xi_G = +0.15$; $\xi_r = 0$. The magnitudes of the radial displacements of the teeth for this gear were determined according to the calculated form[2] of the deformed flexible gear (*ie* disregarding the deformation of the flexible gear under load). A comparison was made for the loads $M_{tor} = 20$; 40 ; 80 ; 100 kg m (the calculated load for the gear was equal to 100 kg m).

Fig 3a&b show the theoretical curves $P(\varphi)$ (indicated by the unbroken line 1) calculated according to the relationship (6) and the experimental curves plotted from the data[3] of the laboratory of the chair 'Machine Parts' of the N.E. Bauman Higher Technical College, Moscow (indicated by the dotted line 2).

Note: When plotting curve 1 (Fig 3) one can also consider that under load the large axis of the flexible gear is displaced relative to the large axis of the generator by 5 to 15° in the direction counter to the rotation of the generator[3]. The calculation is effected by displacing to the left (Fig 3) the calculated calculated curve 1 along the axis $\varphi \sim$ by 5 to 15°.

Comparison of the theoretical and experimental curves has shown that the relationship (6) gives values which are in good agreement with the experiment. The table gives the results of a comparison of the maximum loads on the tooth and the value of the coefficients of non-uniformity of the load distribution over the teeth k .

With a small load ($M_{tor} = 20$; 40 kg m) the deformed flexible gear changes its initial shape insignificantly, and calculation of the geometry of the flexible gear from the static parameters of its deformation has a small error (up to 7%).

As the load increases (M_{tor} = 80 ; 100 kg m and its value approaches that calculated, the shape of the flexible gear becomes distorted, the influence of the total deformation of the system increases, and the calculation of $P(\varphi)$ from the static parameters of the geometry leads to an increase in the error (15%; 21%). On the basis of the theoretical and experimental investigations made the following conclusions may be drawn:

(1) The relationship (6) correctly reflects the connection of $P(\varphi)$ with the basic parameters of the harmonic drive and may be made the basis of the approximate methods. The load acting on the tooth of the harmonic drive, is proportional to the product of the radial displacement of the tooth S_{z_i} on the kinematic function $f(\alpha)$ (deformation of the system is disregarded here).

(2) Distribution of the load between the teeth of the harmonic drive is determined by the shape assumed by the flexible gear after application to it of the operating torque.

(3) If the shape of the deformation of the flexible gear under load is unknown, the distribution of the load between the teeth may be obtained approximately from the calculated shape of the deformation of the flexible gear and the relationship (6). The method for determining $P(\varphi)$ is given below. For $M_{tor} \geqslant 0.8 M_{tor_{cal}}$, the computed value P_{max} should be reduced by ~15%.

(4) The present method is not dependent upon the choice of the driving and driven unit. Any gear may be reduced to the arrangement in question by the inversion method.

(5) The relationship (6) may be also used for determining the load acting on teeth of harmonic gears with free-wave generators, if the shape which the flexible gear assumes under load, is known.

METHOD OF DETERMINATION

given: (a) the geometric parameters of the transmission;

(b) the static shape of the deformation of the flexible gear.

Sequence of the calculation:

(1) Determine (with the step $\Delta\varphi = 5^o$ for small transmissions, and $\Delta\varphi = 3^o$ for large transmissions) the values of the radii-vectors ρ_φ, drawn from the centre of rotation of the deformed flexible gear to points located on the curve which describes the apices of its teeth.

(2) Determine the values $\Delta\rho = \rho_{(\varphi - \Delta\rho)} - \Delta\rho_{(\varphi + \Delta\varphi)}$ for various angles φ .

(3) Construct on an arbitrary scale the graph $\Delta\rho_{\varphi}$ from the angle of rotation of the generator (Fig 4).

(4) Calculate the values for the kinematic function $f(\alpha) = (tg\alpha - tg\alpha_i)$ for the points

$$A(\varphi = 0)\ ; \qquad f(\alpha)_A = tg\alpha\ ;$$
$$C(\varphi \approx 45^o)\ ; \qquad f(\alpha)_c = (tg\alpha - 0.5)\ ; \qquad \text{for } \omega = \Delta \cos 2\varphi\ ;$$
$$D(\varphi = \varphi_{eng})\ ; \qquad f(\alpha)_D \approx 0\ .$$

(5) Construct in an arbitrary (along the vertical) scale on Fig 4 the graph $f(\alpha)$ by joining the straight lines of the point A and C , C and D .

(6) Determine the number of pairs of teeth n located in the active zone of engagement

$$n = \frac{\varphi_{eng} z_r}{2\pi}\ .$$

(7) In Fig 4 separate the abscissa- section $(0 \div \varphi_{eng})$ into n equal parts.

(8) Measure the ordinates, corresponding to the points of division: $[\Delta\rho]_i$, mm ; $[f(\alpha)]_i$, mm ; where $i = 1 \div n$.

(9) Calculate the corresponding products $\left\{[\Delta\rho]_i . [f(\alpha)]_i\right\}$.

(10) Determine $\sum_{i=1}^{i=n} \left\{[\Delta\rho]_i [f(\alpha)]_i\right\}$.

(11) Calculate the value of the scale coefficient

$$h^* = \frac{2M_{tor}}{uD_r \sum_{i=1}^{i=n} \left\{[\Delta\rho]_i [f(\alpha)]_i\right\}}\ .$$

(12) Determine the peripheral forces acting on the teeth of the harmonic drive,

$$P(\varphi)_i = h^* \left\{[\Delta\rho]_i [f(\alpha)]_i\right\}\ .$$

Table 1

M_{tor}, kGm	$\frac{P_{max_{theoret}}}{P_{max_{exper}}}$,	$k_{theor} = \left(\frac{P_{max}}{P_{av}}\right)_{theoret}$	$k_{exper} = \left(\frac{P_{max}}{P_{av}}\right)_{exper}$
20	1.07	1.86	1.86
40	0.98	1.86	1.85
80	1.15	1.86	1.84
100	1.21	1.86	1.97

REFERENCES

No.	Author	Title, etc
1	M.N. Ivanov	The kinematics of harmonic drives. Izvest. VUZ. Mashinostroenie, 8 (1968)
2	S.A. Shuvalov *et al*	Method of calculating the geometry of the engagement of harmonic toothed drives. Izvest. VUZ. Mashinostroenie, 9 (1969)
3	V.A. Finogenov	The load distribution over the teeth of a harmonic drive. Izvest. VUZ. Mashinostroenie, 12 (1970)

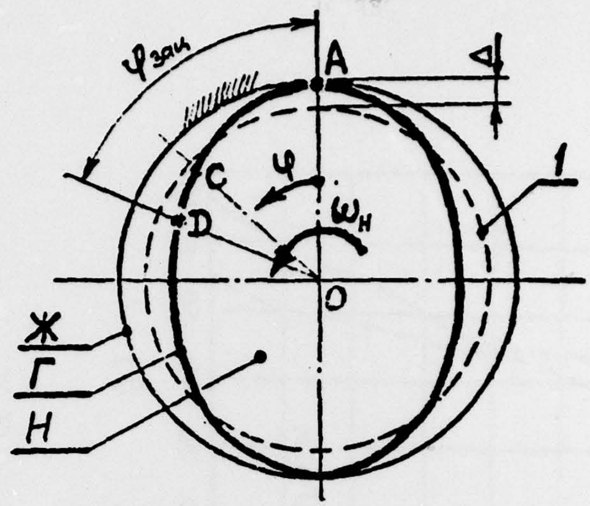


Fig 1 Diagram of a harmonic drive: 1 - flexible gear before deformation

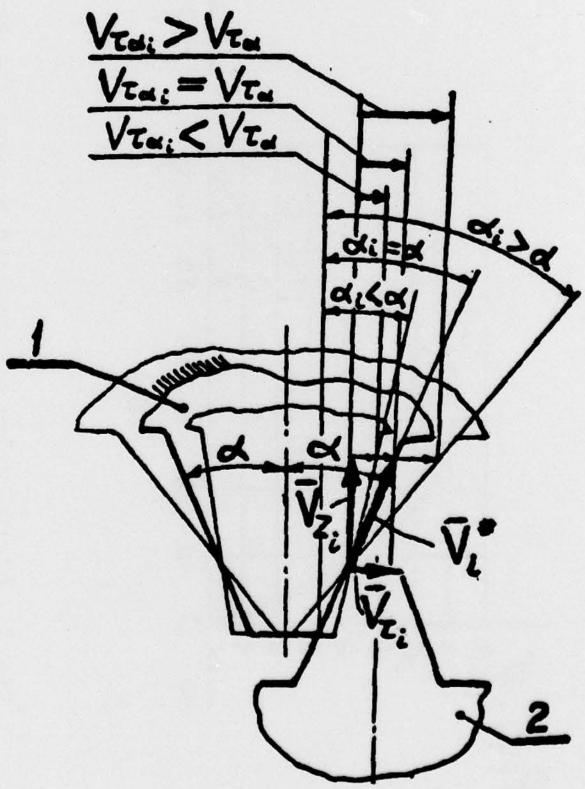


Fig 2 The determination of the velocity of the elastic deformation of a tooth: 1 - tooth of a rigid gear; 2 - tooth of a flexible gear

LT 1929

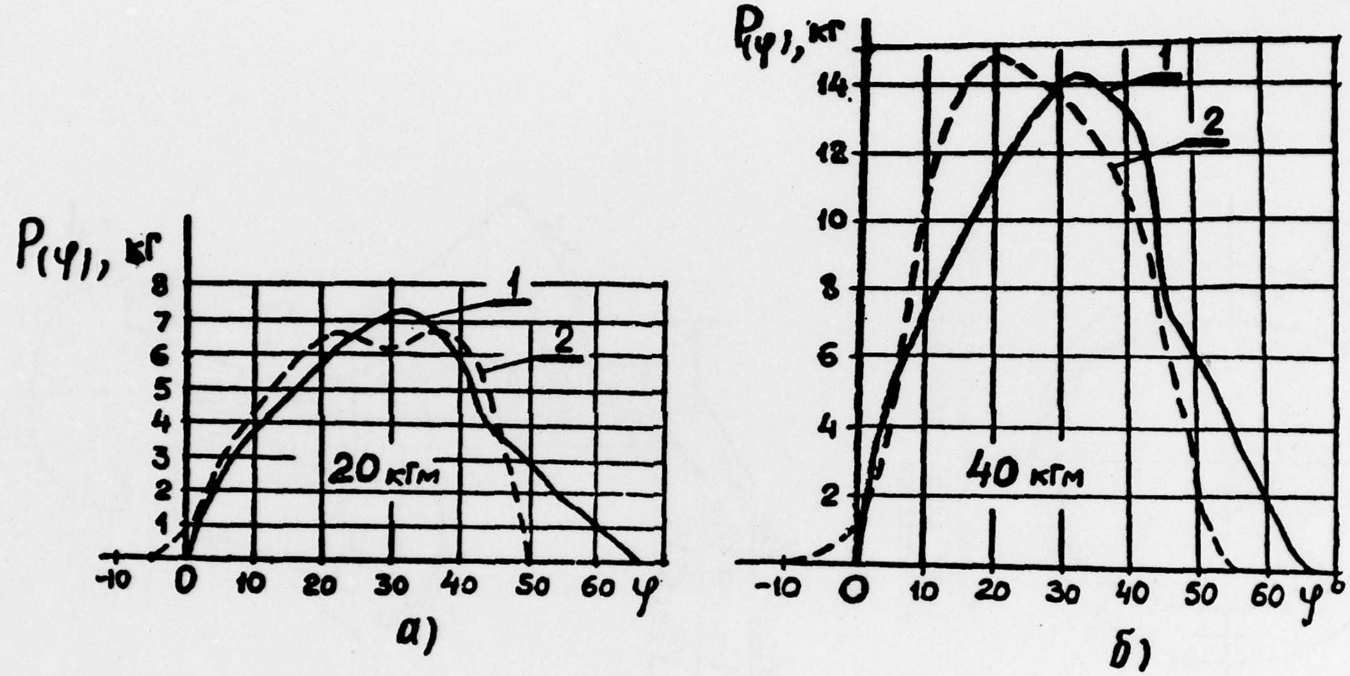


Fig 3 Distribution of the load between the teeth of a harmonic drive: a - M_{tor} = 20 kGm; G - M_{tor} = 40 kGm; 1 - theoretical curve (obtained from the relationship (6)); 2 - experimental relationship

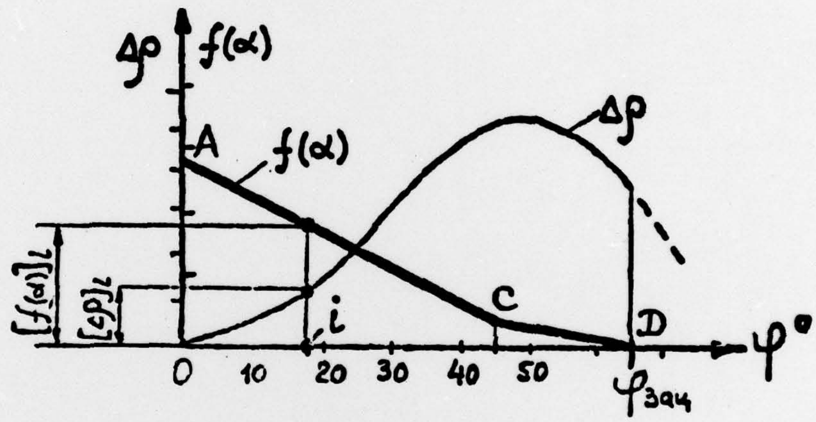


Fig 4   On the method of determining $P(\phi)$

Library Translation 1929

ADVANCE DISTRIBUTION:

RMCS	
ITC	
DRIC	70
HSD, Hatfield	
DOAE	
Library, NGTE	

RAE

Director	
DD(A)	
Engineering Physics Library	10
Main Library	